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BUCKLING OF STEEL COLUMNS WITH ECCENTRIC BRACING

MOHAMED A. ABOKIFA, SHERIF M. IBRAHIM, ABDELRAHIM K. DESSOUKI

Abstract- This paper investigates the behavior and the reduction in the column axial buckling capacity due to the eccentricity in the vertical bracing connection. Vertical bracing system is usually connected concentrically to web of I-shape steel columns. However, due to some constructional limitations, this system is constructed eccentrically. Many of current specifications such as AISC does not define a specific value for the flexural out-of-plane or torsional buckling factors for such cases. A non-linear finite element analysis is used to simulate the eccentric vertical bracing connection to the column web. A wide range parametric study for such case is presented. Various parameters are examined including the eccentricity value, columns cross-section, column slenderness ratio, and location of the bracing joint along column height. The results of this research are presented to quantify the influence of each parameter on the reduction in the column axial capacity due to the eccentricity in vertical bracing connection. A simplified design model to predict the axial buckling capacity including the value of vertical bracing eccentricity is proposed in this paper.

Index Terms- Out-of-plane buckling, Torsional buckling, Finite element analysis, Eccentric bracing, Vertical bracing.

1 INTRODUCTION

Codes and specifications provide a simplified way to predict column capacity by using design equations. AISC 2010 [1] column design equations were developed as a reasonable lower bound to over 300 column test results (Tide, [2]). The possible way to differentiate between the column strength categories is by using the concept of multiple column curves, such as those that were made by Lehigh University (Bjorhovde [3]) and those that were developed by European researchers (Beer and Shultz [4]). State-of-the art column design formulas must be based on extensive studies of maximum strength of representative geometrically imperfect columns (Ziemian [5]).

The out-of-plane buckling strength of column depends on the strut members of the vertical bracing system which are usually attached to the middle of the web of the I-section columns. The connection between those struts and the column web are constructed as simply supported joint, which provides lateral restrain to the column. In ideal and most common situation the strut members can be represented as a pinned lateral support at the center line of the column. If the strut maintains the required ideal stiffness to provide full lateral restraint to the column, the out-of-plane buckling factor will be equal to unity. In certain situations, because of some constructional conditions, the connection between the strut in the lateral bracing system and the web of the I-section column is constructed eccentrically. The current Egyptian code of practice for steel construction, ECP-205 [6] and some other specifications such as AISC [1] do not define specific values for the out-of-plane buckling factors for such cases and most designers take this value by default as unity. The eccentricity of the connection of the strut member will affect the out-of-plane buckling mode and changes it from pure flexural buckling about minor axis to a combination of torsional and flexural buckling. In such cases, the equations provided in ECP-205 [6] and AISC [1] specification will not be applicable.

El-Banna et al. [7] studied the effect of vertical bracing eccentricity on the out-of-plane buckling of H-shape steel columns. They used linear finite element analysis to perform a parametric study on seven sets of H-shaped columns from HEA 400 to HEA 1000. The parametric study results showed clearly that there was a reduction in the critical buckling load due to the eccentricity of vertical bracing connection and this reduction is directly proportional to the column depth. This reduction can reach 30% of the critical load compared to the case of no eccentricity.

Helwig and Yura [8] studied the torsional buckling behavior of wide flange doubly symmetric columns with lateral bracing attached at different locations along the column cross-section. They used finite element analysis to determine the column's torsional capacity as well as the required torsional stiffness for the lateral bracing to make the torsional capacity (P_t) equal to the minor flexural capacity of column (P_{cry}). The results of this study showed that; the torsional buckling capacity of columns with eccentric lateral bracing decreased as the distance between the bracing location and centroid "e" increased. It was required to provide a stiffer torsional brace in case the lateral bracing is attached eccentrically. In addition, the torsional bracing is not effective when the lateral bracing is attached away from the centroid of the cross-section. The main objective of this research is to investigate the behavior of I-section steel columns with eccentric vertical bracing and their failure modes as well as to determine the reduction in the axial capacity for these columns, introduce a practical solution to

- *Mohamed A. Abokifa is an assistant lecturer in structural engineering in Ain Shams University, Cairo, Egypt.* *E-mail: eng.mohamedamin_90@hotmail.com*
- *Sherif M. Ibrahim is an associate Professor in structural engineering in Ain Shams University, Cairo, Egypt* *E-mail: sherif_ibraim@eng.asu.edu.eg*
- *Abdelrahim K. Dessouki is a Professor in structural engineering in Ain Shams University, Cairo, Egypt.* *E-mail: abdelrahim_dessouki@eng.asu.edu.eg*

enhance the eccentric connection of vertical bracing to compensate the reduction in the axial capacity and propose a design procedure in terms of new design formulas to predict the axial capacity of columns with eccentric vertical bracing. A verified non-linear finite element model is adopted to perform a wide range parametric study. This parametric study evaluates the effect of the variation of eccentricity ratios of the bracing joints on the out-of-plane axial capacity for different types of columns cross-sections, various locations of the bracing joints along column length and different steel material grades.

2 FINITE ELEMENT MODEL

The finite element method as described by Zienkiewicz and Taylor [9] has been proven to be very efficient to simulate such cases. In this research, the non-linear finite element analysis of the hot-rolled I-shaped columns is carried out by using ANSYS 14.5 [10] finite element computer program. A detailed description of the used finite element model is provided in the following section.

2.1 Description of The Proposed Model

Shell elements are used to simulate the components of the I-section columns as well as the used column end plates in a three-dimensional model. In this research, four-node thin shell elements "SHELL 181" were used to model the investigated columns. "SHELL 181" is suitable for analyzing thin to moderately thick shell structures, it has both membrane and bending capabilities for the three dimensional analysis of structures. It is a four node element with six degrees of freedom at each node: translations in the x, y and z directions, and rotations about the x, y and z axes. "SHELL 181" is also suitable for linear analysis as well as large rotations and large strain non-linear applications and it also takes into account the change in shell thickness in non-linear analysis. The geometry, node locations and the coordinate system for this shell element are shown in Fig. 1.

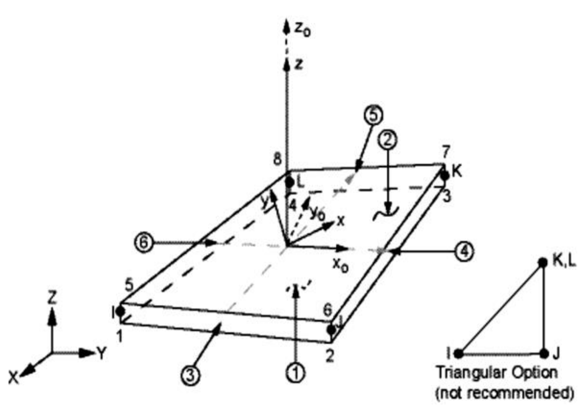
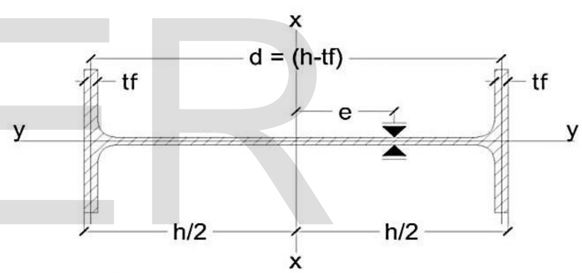


Fig. 1. "SHELL 181" Geometry

The mesh size used in the finite element model is adopted to provide accurate results and less analysis time as well as maintaining a reasonable aspect ratio. The thickness of the two attached end plates at both ends of the column is 30 mm to ensure uniform distribution of the axial stress at the load application points. A central node in the upper end plate is restrained from translation in two horizontal directions (Ux and Uz) in addition to rotation about the column longitudinal axis (Ry), while this end is free to move in the vertical direction in order to allow the axial deformation due to load application. On the other hand, a central node in the lower plate is restrained from translation in the three directions (Ux, Uy and Uz) in addition to rotation about the longitudinal axis (Ry). At the lower end plate all nodes on the line representing the web of the column are restrained from translation in the vertical direction (Uy) in order to overcome the stress concentration in the column's web which may lead to local buckling failure. At mid-length of the column, a node is restrained from translation in the horizontal direction (Ux). This point represents the vertical lateral bracing location. This location of the intermediate lateral bracing is varied along the web length to have eccentricity ratio (e/d) from "0.0" to "0.5" as shown in Fig. 2.



(e/d) = 0.0, 0.1, 0.2, 0.3, 0.4 or 0.5

Fig. 2. Eccentricity value (e/d)

The axial compressive force is represented as a group of equally concentrated loads distributed along the whole perimeter of the modelled column. Fig. 3 shows a typical schematic drawing for the column loading and restrained points.

Geometrical non-linarites are taken into account; with value of maximum imperfection taken as l/1000, where "l" is half column length.

Material non-linearites are taken into consideration by introducing a bilinear stress-strain curve with Young's modulus of elasticity (E = 210 GPa) and with tangent modulus (E_t = 0.05 E) to account for strain hardening. The value of Poisson's ratio is taken equal to 0.3. The idealized stress-strain curve for steel grade S235 is shown in Fig. 4.

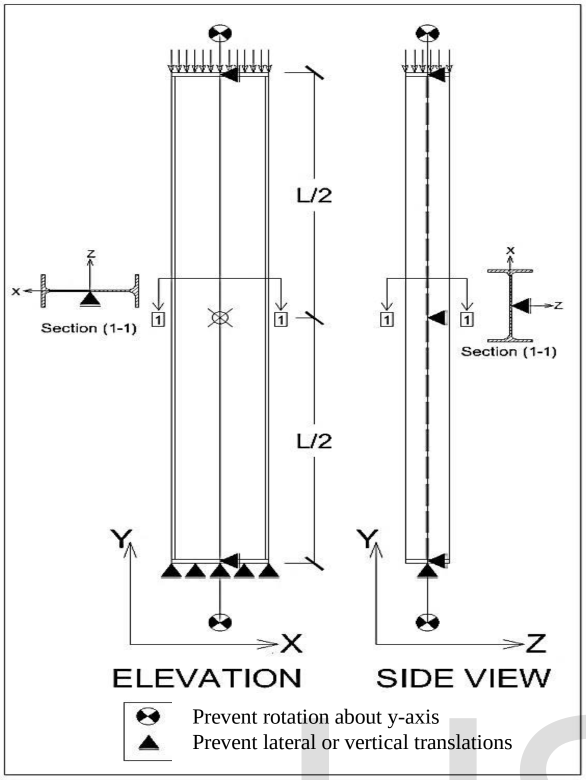


Fig.3. Typical schematic drawing for the column loading and restrained points

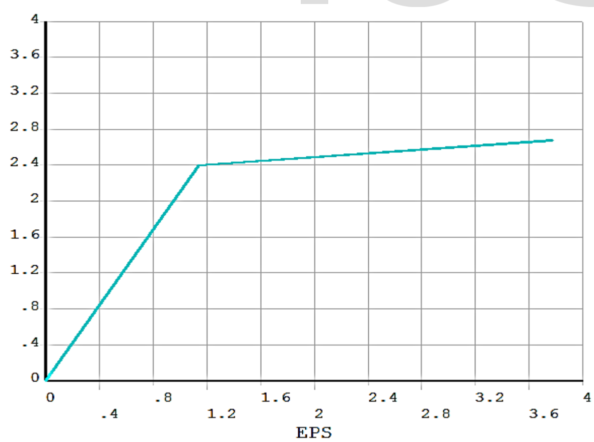


Fig. 4. Idealized stress-strain curve for steel grade S235

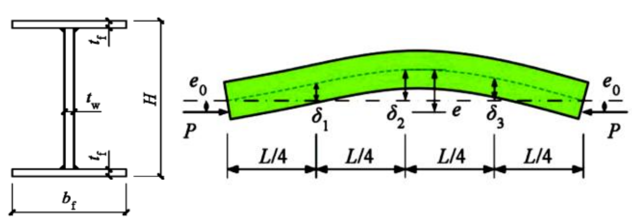
The finite element analysis is conducted through two phases. The first phase is the linear buckling analysis, which is performed on a column having perfect geometry in order to obtain the probable elastic buckling modes of the modelled column. The second phase is the non-linear analysis of the buckled column obtained from the first step. This phase is performed to obtain the ultimate capacity of the initially imperfect column as well as predicting the lateral deformations and failure mode of the examined column. Both material and geometrical non-linearities are incorporated in the non-linear static analysis which is performed by using the Modified Newton-Raphson (MNR) technique and employing the Arc-length control throughout the solution routine of the parametric study.

2.2 Verification of The Proposed Model

The main aim of this part is to verify the accuracy of results obtained from the finite element model. A set of comprehensive experimental works and published numerical results are selected to examine the accuracy of the finite element model.

This section contains verification with three different experimental studies as well as one numerical research. A total of nine different specimens from previous experimental studies and nine different specimens from previous numerical research were used in the verification.

(1) The first experimental research was carried out by Wang et al. [11], who studied strengthened steel columns to investigate the effect of initial load on mechanical properties of steel columns after weld strengthening processes. The control specimen from their work was chosen in the current verification study. The tested member is a long column that failed by flexure buckling about its minor axis. The end constraint of the steel column used single hinged support to maintain the rotation around the weak axis. All the required data about the specimen were reported such as column's full dimensions, the material properties, the yield stress and the modulus of elasticity were measured using tensile coupons. Three measures were done of the initial imperfections along the column length as well as the measures of the eccentric value of the applied axial load. Fig. 5 shows the specimen dimensions and its out-of-straightness values.



L=2999.5 mm, H= 257.5 mm, t_w=9.87 mm, b_f=179.3 mm and, t_f=7.87 mm
δ_1= -0.047 mm, δ_2= -0.223 mm, and δ_3= -0.375 mm
e_{01}= -7.64 mm e_{02}=2.67 mm

Fig. 5. Cross section dimensions and out-of-straightness values of the control specimen Wang et al. [11]

Comparison between results of the experimental study performed by Wang et al. [11] and the results obtained from the non-linear finite element analysis is shown in Table 1. It is obvious that the finite element result is in good agreement with the experimental result with deviation equal to 3.6 %.

TABLE 1

COMPARISON OF FINITE ELEMENT AND EXPERIMENTAL RESULTS OF CONTROL TEST SPECIMEN COLUMN TESTED BY WANG ET AL. [11]

$P_{experiment}$ (ton)	P_{FEM} (ton)	P_{FEM}/ $P_{experiment}$
111.525	107.55	0.964

(2) The second experimental research was carried out by Feng et al. [12], who performed experimental and numerical investigations on high strength steel welded H-section columns. A series of six tests was carried out on different geometries of welded H-section columns fabricated from high strength steel with nominal yield stress of 4.6 t/cm2. Beside their experimental study, Feng et al. [12] conducted finite element analysis on high strength steel welded H-section columns. The finite element modelling was conducted for the tested specimens as well as performing a limited parametric study on different columns cross-sections. A set of nine columns were modelled and examined. The non-linear finite element program used in that study was ABAQUS 6.10. Both linear perturbation analysis and non-linear analysis were performed to obtain the ultimate strengths and failure modes of the high strength steel columns. Both material and geometrical nonlinearities were taken into consideration. However, modelling of residual stresses was not taken into account. Their finite element mesh was varied to provide accurate results as well as less analysis time. Their finite element model takes into consideration all the measured data of the tested specimens such as the measured dimensions and the material properties. The load transfer plates at specimen's ends were modelled using analytical rigid plates. The same end conditions used in the numerical study of the specimens were accurately considered in the modelling. Comparison between results of the experimental and numerical study performed by Feng et al. [12] and the results obtained from the current non-linear finite element analysis are shown in Table 2 in addition to the dimensions of the investigated specimens. It is obvious that the finite element results are in good agreement with the experimental results as well as the numerical results within an average range of deviation 11.2 % for the former and ±1 % for the later.

In conclusion, the results obtained from the finite element model were found to be in very good agreement with the results obtained from previous experimental and numerical studies.

3 PARAMETRIC STUDY

A parametric study is performed to determine the out-of-plane buckling capacity of I-section steel columns using eccentric vertical bracing with respect to their webs. The parametric study can be classified into four main sections in which different parameters are introduced and investigated. The main parameters investigated in this study are: 1) bracing eccentricity ratio (e/d), 2) steel cross section type (IPE, HEB, HEM and HEA), and 3) the steel grade (steel S235 and S355 of yield strength 235 and 355 MPa respectively).

TABLE (2)

COMPARISON OF CURRENT FINITE ELEMENT WITH EXPERIMENTAL & FINITE ELEMENT RESULTS OF HIGH STRENGTH STEEL I-SECTION COLUMNS TESTED BY FENG ET AL. [12]

Specimen number	Feng et al. [12] Exp.	Feng et al. [12] F.E.M	Current F.E.M		
	P_{Exp} (ton)	$P_{FEM (1)}$ (ton)	$P_{FEM (2)}$ (ton)	$P_{FEM(2)}$ / $P_{FEM(1)}$	$P_{FEM(2)}$/ P_{Exp}
L1	162.25	186.49	188.00	1.008	1.158
L2	114.15	129.72	128.53	0.991	1.126
L3	83.95	99.94	98.99	0.991	1.179
L4	212.8	204.62	205.69	1.005	0.966
L5	129.8	147.50	144.93	0.983	1.116
L6	114.3	112.058	107.84	0.962	0.943
L7	N.A	253.48	249.40	0.984	N.A
L8	N.A	191.28	194.70	1.018	N.A
L9	N.A	99.93	102.04	1.021	N.A

The results of the parametric study are shown in the form of graphical relationships. Three main types of graph sets are used in this section. The first set represents the relation between "P_{FEM}/P_y" and the out-of-plane slenderness parameter "Kl/r_y". The second graph set represents the relation between "$P_{e/d=0.5}/P_{e/d=0.0}$" and column cross-section depth "d". The third graph set represents the relation between "P_{FEM}/P_y" and the eccentricity ratio "e/d". These parameters are defined as follows:

- P_{FEM}: maximum nominal strength of the investigated columns obtained from non-linear finite element analysis.
- P_y: yield strength of the column.
  Where: $P_y=A_{total} \times F_y$ (1)
  A_{total}: is the total area of the column cross-section,
  F_y: is the yield stress of the used steel.
- Kl/r_y: is the out-of-plane slenderness parameter of the column.
  K: is the effective length factor and its value is usually taken equal to unity,
  l: is half the column length,
  r_y: is the column cross-section minor radius of gyration.
- $P_{e/d=0.5}$: is the maximum nominal strength obtained from non-linear finite element analysis of the column which has eccentric vertical bracing attached to their flange.
- $P_{e/d=0.0}$: is the maximum nominal strength obtained from non-linear finite element analysis of the column which has centric vertical bracing attached at the middle of its web.

3.1 Effect of Eccentricity Ratio (e/d):

The eccentricity ratio in this part of study varied from "zero" to "0.5" with increment "0.1". The study is performed on seven cross-sections namely; IPE600, IPE400, IPE 300, HEB 1000, HEB 800, HEB 700 and HEB 500. All specimens have pin-ended boundary conditions and lateral bracing is located at mid-length (i.e. a/L=0.5). Various out-of-plane slenderness ratios (Kl/r_y) of 60, 70, 80, 100, 120, 140, 160, 180 and 200 were considered. The material considered in this study is steel grade S235 with yield strength 235 Mpa. Fig. 6 and 7 show samples of the relationship between "P_{FEM}/P_y" and "Kl/r_y" for IPE 600 and IPE 300 columns respectively. The most common trend detected in those graphs is that the ultimate axial capacity of the columns decreases with the increase of the eccentricity value (e/d). This happens due to the difference between the developed buckling shapes of the columns which have concentric vertical bracing to their webs and those having eccentric vertical bracing. The effect of using eccentric vertical bracing to column web is significant for short columns or in another words; columns with small out-of-plane slenderness values (Kl/r_y). On the other hand, this effect is not recognized for long "elastic" columns. This is because the torsional buckling is dominant for short columns while flexural buckling about minor axis is dominant for long columns. It is clear that the small eccentricity ratio values such as (e/d=0.1, 0.2), the ultimate axial capacity of columns is not affected by such eccentricity ratio values. This is observed for IPE 600 columns, while for IPE 300 the eccentricity ratios (e/d=0.1, 0.2 and 0.3) will not affect the column axial capacity. The main reason for this is that for small eccentricity ratios (e/d) the column undergoes flexural buckling about its minor axis as the vertical bracing will prevent lateral translation as well as twisting of the column cross-section. It is observed that the maximum drop in the axial capacity occurs for columns with out-of-plane slenderness ratio (Kl/r_y) equal to 80. The maximum drop in axial capacity of IPE 600 and IPE 300 columns with eccentricity ratio (e/d=0.5) is 29% and 16% respectively.

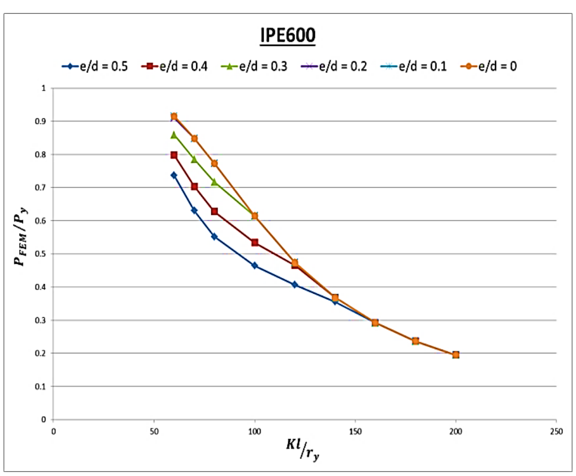


Fig. 6. P_{FEM}/P_y versus Kl/r_y for IPE 600 column supported at mid-length for different e/d ratio.

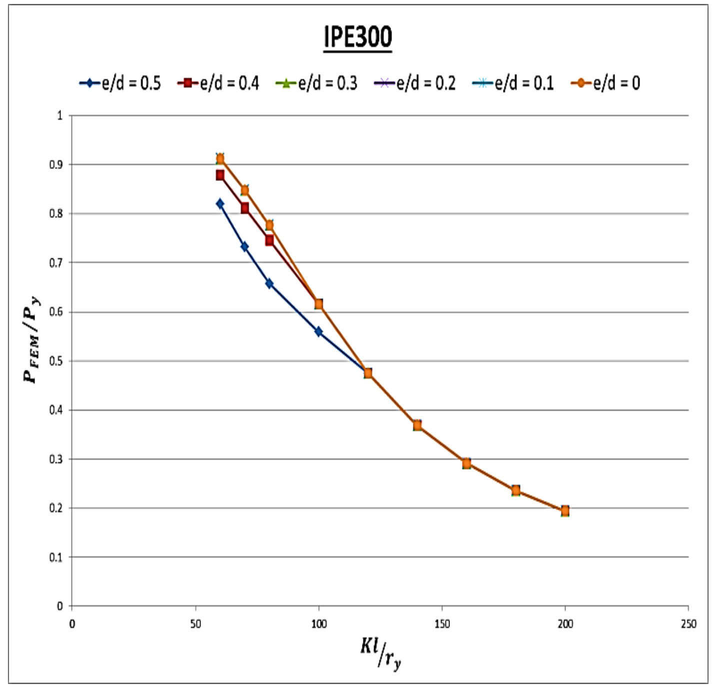


Fig.7. P_{FEM}/P_y versus Kl/r_y for IPE 300 column supported at mid-length for different e/d ratios

Fig. 8 and 9 show the two different buckling shapes for two e/d ratios equal to 0 and 0.5 respectively. The first buckling mode shape in Fig. 8 represents the ideal case for a pinned-end column which is laterally supported at its mid-length and its mid-web. For this case the column undergoes pure global buckling about its minor axis. The ultimate capacity of this column can be well predicted by the nominal strength equations for the compression members specified in chapter "E" in AISC [1] specifications.

The second buckling mode shape in Fig. 9 represents the case in which the column has eccentric vertical bracing to its web with eccentricity ratio (e/d = 0.5). It is clear that the buckling shape has changed from the first case and it tends to be a combination between lateral and torsional buckling along the total length of the column. Consequently, the column with eccentric bracing, cross-section will rotate around the laterally supported point to develop a buckling shape similar to that of the singly symmetric columns. The vertical bracing joint prevents lateral translation only while the torsional buckling of the column may govern its axial capacity and would be smaller than flexural buckling about its minor axis.

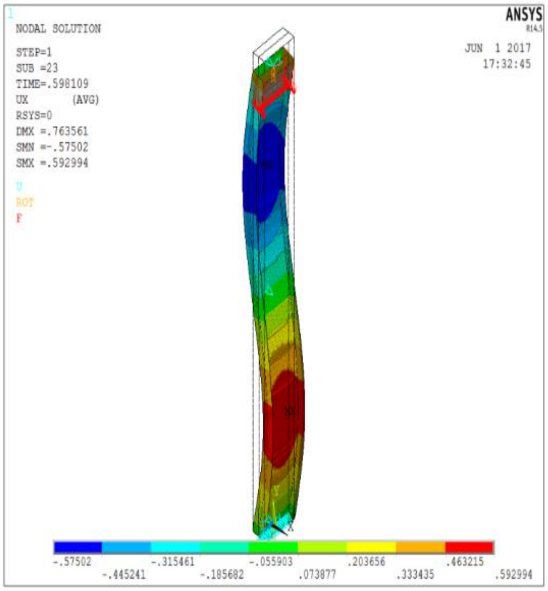


Fig. 8. Buckling shape of IPE 600 column with (Kl/r_y=80) and supported at mid-length with concentric vertical bracing

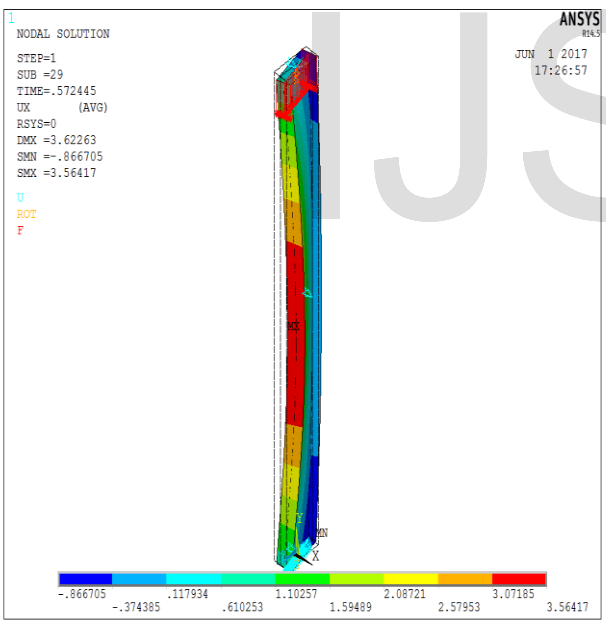


Fig. 9. Buckling shape of IPE 600 column with (Kl/r_y=80) and supported at mid-length with eccentric vertical bracing with eccentricity ratio (e/d=0.5)

Fig. 10 and 11 show the relationship between "P_{FEM}/P_y" and eccentricity values "e/d" for IPE and HEB cross sections used in columns with out-of-plane slenderness ratio (Kl/r_y=80). It is evident from these figures that as the depth of column cross-section increases, the drop in column capacity with eccentric vertical bracing increases. For IPE 600 column the maximum drop in capacity is 29%. On the other hand, the maximum drop in capacity for IPE 300 is 16%. For a given (e/d) ratio, as the depth of column increases, the eccentricity value (e) of bracing will increase and this will lead to larger drop in the axial capacity. That is why larger column depth is usually accompanied with larger drop in the axial capacity. It is also evident from these graphs that the effect of using eccentric vertical bracing is significant with columns which have a bigger cross-section than HEB 500, while this observation is not recognized with IPE columns. IPE cross sections with smaller depths (e.g. IPE 300) are affected by the eccentricity in vertical bracing.

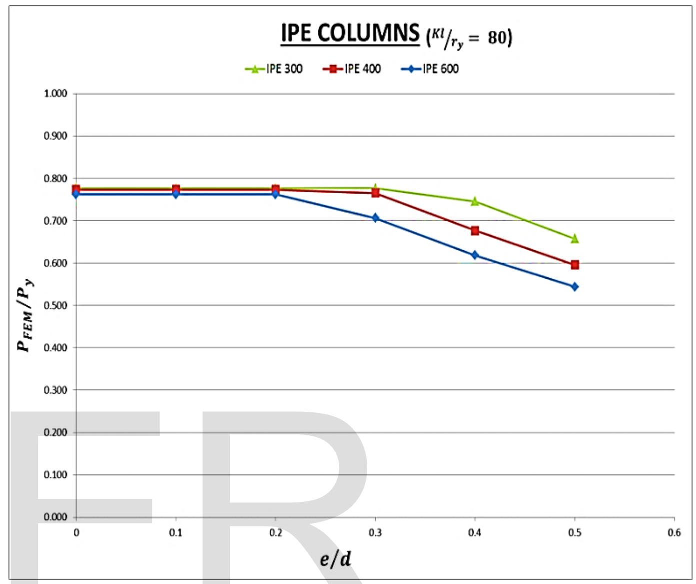


Fig.10. P_{FEM}/P_y versus e/d for set of IPE columns with (Kl/r_y=80) and supported at mid-length

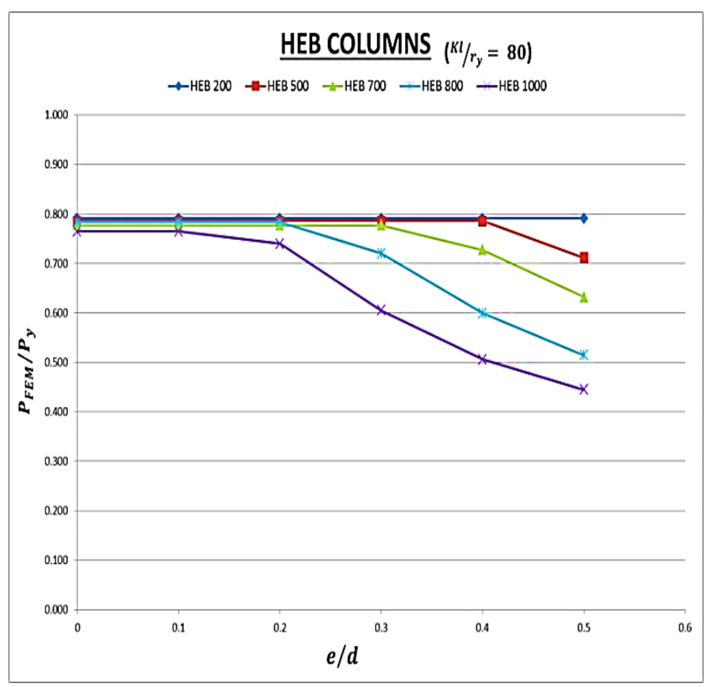


Fig. 11. P_{FEM}/P_y versus e/d for set of HEB columns with (Kl/r_y=80) and supported at mid-length

3.2 Effect of Using Different Cross-Section Types

To investigate the effect of using different hot rolled cross sections on the drop in the axial capacity when using eccentric vertical bracing, another study is performed on a number of different types of hot rolled cross sections namely IPE, HEM, HEB and HEA with out-of-plane slenderness ratio (Kl/r_y = 80) and eccentricity ratios (e/d) equal to 0.0 and 0.5 respectively. Steel grade S235 is used in this study. The results of this study are represented in Fig. 12. It is clear that the axial capacity of the columns with eccentric vertical bracing decreases with the increase of the depth of the column cross-section. For any column depth, the drop in the axial capacity for columns with high torsional rigidity is smaller than that of columns with small torsional rigidity. In other words, IPE cross sections have the maximum drop of out-of-plane axial capacity with eccentric bracing amongst all studied cross sections while HEM cross sections has the least drop. A comparison between HEB 400 and IPE 400 columns is done in order to verify the difference in behavior resulting from changing the type of column cross-section. As evident from Fig. 12, HEB 400 has its full out-of-plane axial capacity at e/d=0.5 while an IPE 400 has 23% reduction of the out-of-plane axial load capacity at e/d =0.5. This main difference between these types of sections can be attributed to the ability of HEB section to resist twisting as well as out-of-plane flexural buckling. The HEB 400 column which has a bigger rigidity for twisting is not greatly affected by using eccentric vertical bracing.

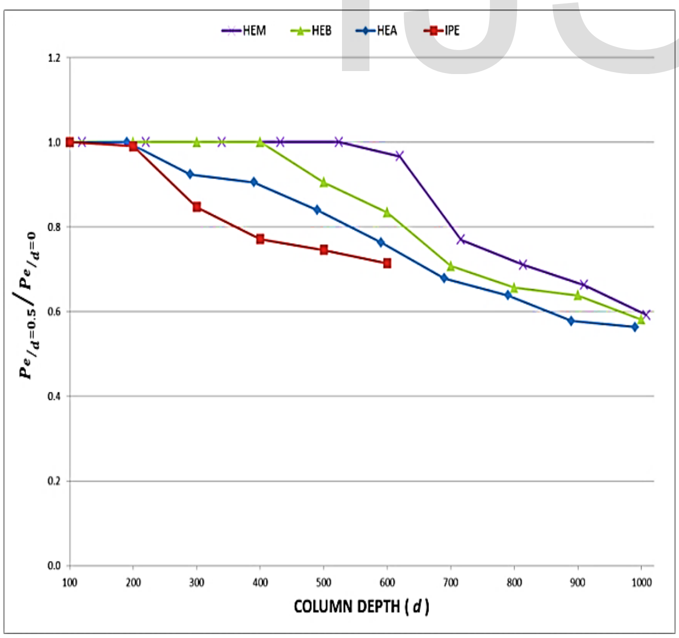


Fig. 12. $P_{e/d=0.5}/P_{e/d=0}$ versus columns depth with (Kl/r_y=80) and supported at mid-length for different hot rolled cross sections

3.3 Effect of Steel Grade

Fig. 13 shows the relationship between "P_{FEM}/P_y" and "Kl/r_y" for IPE 600 cross section with steel grade S355 with nominal yield stress (F_y= 355 MPa). This study is conducted on columns with vertical bracing joint attached at the column mid-length. It is clear that, the maximum drop in the axial capacity for columns with higher steel grade is bigger than that of columns with steel grade S235 that is shown in Fig. 6. For example, the maximum drop in the axial capacity for IPE 600 from steel grade S355 is 40% while drop for the same cross section from steel grade S235 is 29%. In addition, columns with higher steel grade will be subjected to a drop in the axial capacity even when using eccentric bracing with small eccentricity ratios. The main reason for the increase of the drop in the axial capacity for columns with higher steel grade is that the maximum drop in axial load occurs at the inelastic buckling region in which the capacity of columns with concentric bracing is proportional to yield strength.

4 PROPOSED DESIGN MODEL

Equations provided in chapter "E" of the AISC [1] specifications can be used to predict the axial capacity of columns with concentric vertical bracing. However, in this research, those equations are not applicable to be used with this non-standard structural problem. These equations should be modified to account for the eccentricity of the column lateral bracing.

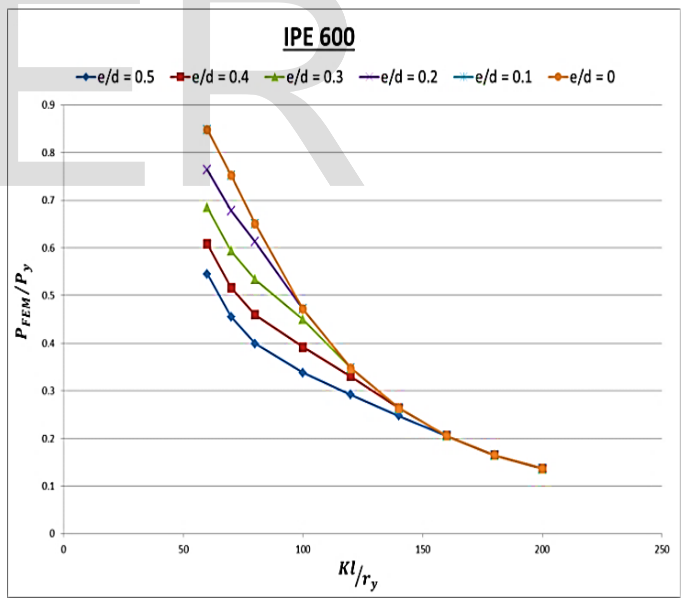


Fig..13 "P_{FEM}/P_y" versus "Kl/r_y" for IPE 600 column supported at mid-height with steel grade S355

A proposed design procedure and new formulas are introduced in this section which can be applied to hot rolled IPE and HEB column cross sections. Commonly, for I-section columns with concentric vertical bracing, the dominant mode of buckling is flexural buckling about the minor axis. However, for columns with eccentric vertical bracing there are two possible buckling modes which are: a) flexural buckling about the minor axis or b) torsional buckling. The lower axial capacity between two will be the governing one in the design. Nominal compressive load for flexural buckling about the minor axis (P_{ny}) is calculated as given in AISC [1]. However, the calculation of the nominal compressive load for torsional buckling (P_{nz}) in case of eccentric bracing is modified as described below. Firstly, the elastic torsional buckling stress (F_{ez}) is calculated as follows:

$$F_{ez} = \left(\frac{\pi^2 E\, C_W}{(K_z\, L)^2} + GJ\right)\frac{1}{A_g\, r_o^2} \qquad (2)$$

For I-sections; $C_W = I_y \frac{d^2}{4}$ (3)

By replacing "C_W" by "$I_y \frac{d^2}{4}$" in (2)

$$F_{ez} = \left(\frac{\pi^2 E\, I_y \frac{d^2}{4}}{(K_z\, L)^2} + GJ\right)\frac{1}{A_g\, r_o^2} \qquad (4)$$

Where G is shear modulus, J is the torsional constant, A_g is the cross sectional area, and d is the column depth.

The term $\left(\frac{\pi^2 E\, I_y}{(K_z\, L)^2}\right)$ in (4) can be replaced by "P_{ey}", where "P_{ey}" represents the elastic flexural buckling load calculated based on a length equals to (K_z L), thus F_{ez} can be rewritten as follows:

$$F_{ez} = \frac{P_{ey}\,(d^2/4) + GJ}{A_g\, r_o^2} \qquad (5)$$

where, r_o^2 = polar radius of gyration about the shear center.

$$r_o^2 = X_0^2 + Y_0^2 + r_x^2 + r_y^2 \qquad (6)$$

X_o, Y_o = coordinates of the shear center with respect to the centroid.

For I-section columns ($X_o, Y_o = 0$) as the shear center coincides with the centroid of the cross-section. For the case of I-section columns with eccentric vertical bracing, the column undergoes torsional buckling. The cross-section of the column will rotate around the braced point. Thus, center of rotation location can be proposed as follows: ($X_o = 0$) while, ($Y_o =$ "e" the distance between centroid and the location of the braced point).

By replacing "r_o^2" by "$e^2 + r_x^2 + r_y^2$" in (5)

$$F_{ez} = \frac{P_{ey}\,(d^2/4) + GJ}{A_g(e^2 + r_x^2 + r_y^2)} \qquad (7)$$

The torsional unbraced length (K_z L) used in (P_{ey}) is the distance between the points on the column which is braced against torsion. Columns with concentric vertical bracing have ($K_z = 0.5$), where the unbraced length for torsion is equal to half length of the column. However, columns with eccentric vertical bracing have ($K_z > 0.5$). A proposed formula is used to calculate "K_z".

$$K_z = 0.5\,(1 + \alpha\,\frac{e}{d}) \qquad (8)$$

Where; K_z= the torsional unbraced length factor for I-section columns with eccentric vertical bracing.

α = factor depends on the type of the column cross-section and the depth of the column cross-section.

$$\alpha_{HEB} = 1.1\left[\left(\frac{h_{HEB}}{1000}\right)^2 + \left(\frac{h_{HEB}}{1000}\right)^4\right] \quad \leq 1.1 \qquad (9)$$

Where; h_{HEB}= column cross-sectional depth in "mm",

$$\alpha_{IPE} = 0.4\left[\left(\frac{h_{IPE}}{600}\right)^2 + \left(\frac{h_{IPE}}{600}\right)^4\right] \quad \leq 0.4 \qquad (10)$$

Where; h_{IPE}= column cross-sectional depth in "mm"
Based on the above proposal, the elastic torsional buckling stress in 7 takes into considerations the value of vertical bracing eccentricity "e", cross section shape and size as well as the eccentricity-to-depth ratio (e/d). The proposed formulas for "K_z"is chosen carefully to be best fitted with the results of the finite element analysis. The values of "F_{ez}" in 7 can be compared to yield strength "F_y" as specified in AISC [1] to determine whether the buckling mode is elastic or inelastic and nominal compressive load "P_{nz}" can be calculated. Table 3 shows comparison between the results of the finite element model and the proposed model for two cross sections IPE 600 and HEB 1000 with eccentricity ratio (e/d =0.5) with steel grade S235. It is evident that there is excellent agreement between the proposed design model and the finite element results. Detailed comparison for other cross sections and different eccentricity ratios is provided by Abokifa [13].

TABLE (3): COMPARISON BETWEEN FINITE ELEMENT MODEL AND PROPOSED DESIGN MODEL FOR E/D=0.5 AND STEEL S235

Cross Section	Kl/r_y	P_{FEM} (ton)	Proposed Design Model			P_{design}/ P_{FEM}
			P_{ny} (ton)	P_{nz} (ton)	P_{design} (ton)	
IPE 600	80	200.24	267.06	208.69	208.69	1.04
	120	147.36	182.28	143.20	143.20	0.97
	160	105.93	108.87	109.07	108.87	1.03
	200	70.63	69.68	93.24	69.68	0.99
HEB 1000	80	474.04	690.46	456.02	456.02	0.96
	120	340.80	468.54	325.81	325.81	0.96
	160	276.45	278.56	273.00	273.00	0.99
	200	184.20	178.28	248.55	178.28	0.97

5 CONCLUSIONS

A non-linear finite model has been developed to investigate the effect of vertical bracing eccentricity on the out-of-plane column strength. A simplified design model is proposed to predict the column strength with eccentric vertical bracing. The main conclusions of this paper can be summarized as follows:

1. The effect of using eccentric vertical bracing to column web is significant for short columns, while this effect is not significant for long columns. This is because the torsional buckling is dominant for short columns while flexural buckling about minor axis is dominant for long columns.
2. The maximum drop in the axial capacity of columns with eccentric vertical bracing is recognized in columns with out-of-plane slenderness ratio (Kl/r_y) equal to 80.
3. As the depth of column cross-section increases, the drop in capacity developed from using eccentric vertical bracing increases and the range of columns influenced by this eccentricity increases while the opposite is true.
4. Using cross-sections having high torsional rigidity in case of eccentric vertical bracing is only effective in increasing column capacity for columns with smaller cross-section depths.
5. The maximum drop in the axial capacity for columns with higher steel grade is bigger than that of columns with lower steel grade. In addition, columns with higher steel grade will be subjected to a drop in the axial capacity even when using eccentric bracing with small eccentricity ratios.
6. Simplified design equations for the torsional buckling strength and design procedure are proposed to be applied for the case of using I-section column with eccentric vertical bracing for IPE and HEB cross sections which takes into considerations the eccentricity ratio as well as the member type and size.

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